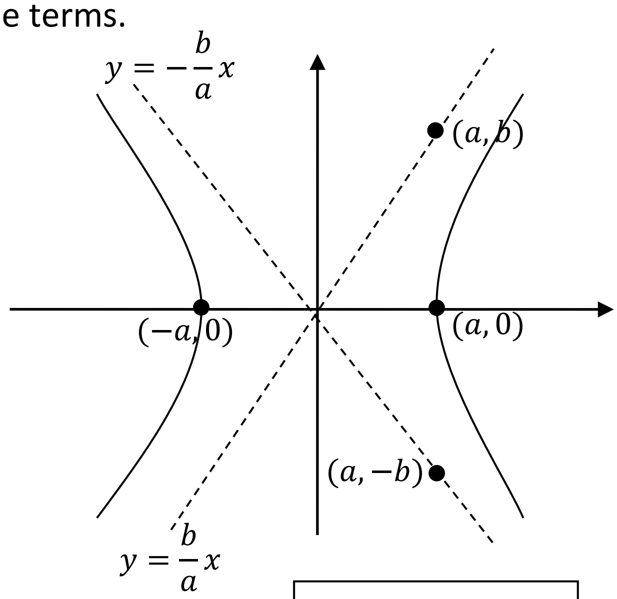
4) HYPERBOLA

The basic equations for a hyperbola are very similar to the equations for the ellipse, but there is a negative sign between the two left hand side terms.

The basic equations for a hyperbola are:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

- Centre = (0, 0)
- x – intercepts: $(-a, 0), (a, 0)$
- Asymptotes: $y = \frac{b}{a}x$, $y = -\frac{b}{a}x$



$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Centre = (h, k)
Vertices are now $+a$ and – a from centre (h, k), not the origin.
Asymptotes are now: $(y - k) = \frac{b}{a}(x - h)$, $(y - k) = -\frac{b}{a}(x - h)$

Quick sketch of asymptotes:
Point 1: centre
Point 2: (a, b)
Point 3: (a, -b)

<u>COMPLETING THE SQUARE FOR A HYPERBOLA</u>

Examples:

1) Sketch $9x^2 - 18x - 4y^2 + 16y - 43 = 0$.

$$9x^2 - 18x - 4y^2 + 16y - 43 = 0$$
$$9(x^2 - 2x) - 4(y^2 - 4y) = 43$$
$$9\left[x^2 - 2x + (\frac{-2}{2})^2\right] - 4\left[y^2 - 4y + (\frac{-4}{2})^2\right] = 43 + 9(\frac{-2}{2})^2 - 4(\frac{-4}{2})^2$$
$$9[x^2 - 2x + 1] - 4[y^2 - 4y + 4] = 43 + 9 \cdot 1 - 4 \cdot 4$$
$$9(x-1)^2 - 4(y-2)^2 = 36$$
$$\frac{9(x-1)^2}{36} - \frac{4(y-2)^2}{36} = \frac{36}{36}$$
$$\frac{(x-1)^2}{4} - \frac{(y-2)^2}{9} = 1$$
$$\frac{(x-1)^2}{2^2} - \frac{(y-2)^2}{3^2} = 1$$

Centre = $(1, 2)$, $a = 2$, $b = 3$
Asymptotes: $(y - 2) = \frac{3}{2}(x - 1) \quad and \quad (y - 2) = -\frac{3}{2}(x - 1)$
simplified: $y = \frac{3}{2}x + \frac{1}{2} \quad and \quad y = -\frac{3}{2}x + \frac{7}{2}$.

2) Sketch $4x^2 - 9y^2 + 8x + 126y - 581 = 0$

$4x^2 - 9y^2 + 8x + 126y - 581 = 0$
$4x^2 + 8x - 9y^2 + 126y = 581$
$4(x^2 + 2x) - 9(y^2 - 14y) = 581$
$4\left[x^2 + 2x + (\frac{2}{2})^2\right] - 9\left[y^2 - 14y + (\frac{-14}{2})^2\right] = 581 + 4(\frac{2}{2})^2 - 9(\frac{-14}{2})^2$
$4[x^2 + 2x + 1] - 9[y^2 - 14y + 49] = 581 + 4 \cdot 1 - 9 \cdot 49$
$4(x+1)^2 - 9(y-7)^2 = 144$
$\frac{4(x+1)^2}{144} - \frac{9(y-7)^2}{144} = \frac{144}{144}$
$\frac{(x+1)^2}{36} - \frac{(y-7)^2}{16} = 1$
$\frac{(x+1)^2}{6^2} - \frac{(y-7)^2}{4^2} = 1$

Centre = $(-1, 7)$, $a = 6$, $b = 4$
Asymptotes: $(y-7) = \frac{4}{6}(x+1) \quad and \quad (y-7) = -\frac{4}{6}(x+1)$
simplified: $y = \frac{2}{3}x + 7\frac{2}{3} \quad and \quad y = -\frac{2}{3}x + 6\frac{1}{3}$.

<u>HOW TO FORM AN EQUATION, GIVEN THE GRAPH OF A HYPERBOLA</u>

1) Start with the basic equation of a hyperbola: either $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$.

2) Pick a point on your hyperbola that is NOT either of the vertices.

3) Substitute the coordinates of that point, and the value of a, into the equation for a hyperbola, and solve for b.

4) Write your final equation.

Worksheet
Delta Ex 37.4 pg 36.7 Q2 – 5, 7 (ignore foci).
Extension: Q6, 8, 9.